Assignment 11.

1. With respect to the origin O, the points P, Q and R have position vectors given by

$$\overrightarrow{OP} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}, \qquad \overrightarrow{OQ} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \qquad \overrightarrow{OR} = \mathbf{i} - 4\mathbf{j} + 5\mathbf{k}.$$

   The point S is such that $\overrightarrow{SP} = \frac{1}{2}\overrightarrow{QR}$.

   (a) Determine the position vector of S. [2]

   (b) Find the unit vector in the direction of $\overrightarrow{OS}$. [2]

2. The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{k} + s(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

   respectively. Show that l and m intersect, and find the position vector of their point of intersection. [5]

3. The lines l and m have vector equations

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

   respectively.

   (a) Show that l and m do not intersect. [4]

   The point P lies on l and the point Q has position vector $2\mathbf{i} - \mathbf{k}$.

   (b) Given that the line PQ is perpendicular to l, find the position vector of P. [4]

(c) Verify that Q lies on m and that PQ is perpendicular to m. [2]

4. With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = 4\mathbf{i} + \mathbf{k}, \qquad \overrightarrow{OB} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}, \qquad \overrightarrow{OC} = \mathbf{i} + \mathbf{j}, \qquad \overrightarrow{OD} = -\mathbf{i} - 4\mathbf{k}.$$

(a) Calculate the acute angle between the lines AB and CD. [4]

(b) Prove that the lines AB and CD intersect. [4]

(c) The point P has position vector $\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. Show that the perpendicular distance from P to the line AB is equal to $\sqrt{3}$. [4]

5. (†) $\triangle ABC$ is given in a three-dimensional space.

(a) H is a point such that $AH \perp BC$ and $BH \perp CA$. Using vector geometry, prove that $CH \perp AB$. [3]

(b) Given the coordinates $A(1, 0, 0)$, $B(0, 2, 0)$ and $C(0, 0, 3)$, determine the locus of all such points H that satisfy both $AH \perp BC$ and $BH \perp CA$. [4]

Total mark of this assignment: $31 + 7$.

The symbol (†) indicates a bonus question. Finish other questions before working on this one.